

# Wave Parameter Definitions

Spoondrift

## Abstract

This document describes definitions of (bulk) spectral variables output by Spoondrift Spotter. In general, our definitions follow standard oceanographic practice, and the specifics are given here. For motivation and/or physical significance of the definitions we refer to the bibliography at the end of this document.

## Document History

Date	Author	Comment
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## Wave spectrum and directional moments

Spotter measures the wave-induced 3D displacements as East-West ( $x(t)$ ), North-South ( $y(t)$ ) and Up-Down ( $z(t)$ ). By default, Spotter samples displacement data at 2.5 Hz for 30 minutes and computes the cross-spectral matrix. The displacement data and the spectral information is stored on the internal storage (sd-card) and the spectra are used to calculate statistical wave parameters (detailed below) that are transmitted to the Spoo-drift Dashboard.

Specifically, spectral estimates of (cross-)spectral densities are provided on a discretely sampled frequency grid

$$f_j = j\Delta f \quad j = 4 \dots 127 \quad (1)$$

where  $\Delta f = f_s/256 \approx 0.0097$  Hz. Note that frequencies below  $f_4 = 0.029$  are not resolved by Spotter and in the analysis this information is not used (and set to NaN in output on internal storage).

For each discrete frequency, Spotter calculates spectral estimates for the following:

Symbol	Spectral Quantity
$E_{zz}(f_j)$	Surface elevation variance density spectrum
$E_{xx}(f_j)$	$x$ -displacement variance density spectrum
$E_{yy}(f_j)$	$y$ -displacement variance density spectrum
$C_{xy}(f_j)$	$x$ - $y$ co-spectrum
$Q_{xz}(f_j)$	$x$ - $z$ quad-spectrum
$Q_{yz}(f_j)$	$x$ - $y$ quad-spectrum

The surface elevation variance density spectrum is used to calculate bulk wave heights and period measures. Directional properties are estimated using the lowest order directional moments of the directional distribution, which are related to the (co/quad-)spectra and displacement spectra as [e.g. Kuik et al., 1988, their appendix A]

$$a_1(f) = \frac{Q_{xz}}{\sqrt{E_{zz}(E_{xx} + E_{yy})}} \quad (2)$$

$$b_1(f) = \frac{Q_{yz}}{\sqrt{E_{zz}(E_{xx} + E_{yy})}} \quad (3)$$

$$a_2(f) = \frac{E_{xx} - E_{yy}}{E_{xx} + E_{yy}} \quad (4)$$

$$b_2(f) = \frac{2C_{xy}}{E_{xx} + E_{yy}} \quad (5)$$

where  $a_n$  and  $b_n$  are the lowest order cosine and sine Fourier series components, respectively.

## Bulk Parameter definitions

To define wave statistical Bulk parameters we will use various spectral frequency moments, defined as

$$m_n = \int_{f_4}^{f_{127}} E(f) f^n df \quad (6)$$

where the lower and upper integration limits coincide with the limits of the variance density spectrum  $E(f)$ . Further, from the directional moments  $a_1, b_1, a_2, b_2$  mean directional moments are defined as

$$\begin{bmatrix} \bar{a}_1 \\ \bar{b}_1 \\ \bar{a}_2 \\ \bar{b}_2 \end{bmatrix} = \frac{1}{m_0} \int_{f_4}^{f_{127}} \begin{bmatrix} a_1(f) \\ b_1(f) \\ a_2(f) \\ b_2(f) \end{bmatrix} E(f) df \quad (7)$$

where  $\bar{a}_1$  (and  $\bar{b}_1$  etc) is the spectrally-weighted mean directional moment. In all cases the integration of frequency-dependent quantities  $F(f)$  is approximated discretely according to the midpoint rule as

$$\int_{f_4}^{f_{127}} F(f) df \approx \sum_{j=4}^{127} F(f_j) \Delta f \quad (8)$$

where  $\Delta f$  is the frequency interval.

### Significant Wave height

The significant wave height  $H_{m_0}$  is estimated from the the variance density spectrum as [e.g. Holthuijsen, 2007]

$$H_{m_0} = 4\sqrt{m_0} \quad (9)$$

with  $m_0$  defined as in equation (6).

### Mean Period $T_{m01}$

A characteristic mean period of the wave field is estimated from the the variance density spectrum according to [e.g. Holthuijsen, 2007]

$$T_{m01} = \frac{m_0}{m_1} \quad (10)$$

with  $m_0, m_1$  defined as in equation (6).

### Mean Direction $\bar{\theta}$

The mean direction  $\bar{\theta}$  is measured clockwise in degrees from North and points in the direction the waves come from. It is obtained from the directional moments as [Kuik et al., 1988, their equation 12]

$$\bar{\theta} = 270^\circ - \frac{180^\circ}{\pi} \arctan2(\bar{b}_1, \bar{a}_1) \quad (11)$$

where  $\bar{a}_1$  and  $\bar{b}_1$  are the lowest order bulk directional moments (see eq. (7)). Note that  $\arctan 2(y, x)$  denotes the multivalued arc-tangent function to resolve the full 360 circle. We ensure that  $0^\circ \leq \bar{\theta} \leq 360^\circ$  by adding/subtracting  $360^\circ$  to angles smaller than  $0^\circ$  or larger than  $360^\circ$ , respectively.

### Mean Directional Spreading $\bar{\sigma}_\theta$

The mean directional spreading is measured in degrees and obtained from the bulk directional moments as [Kuik et al., 1988, their equation 24]

$$\bar{\sigma}_\theta = \frac{180^\circ}{\pi} \sqrt{2 \left( 1 - \sqrt{\bar{a}_1^2 + \bar{b}_1^2} \right)} \quad (12)$$

where  $\bar{a}_1$  and  $\bar{b}_1$  are the lowest-order bulk directional moments (see eq. (7)).

### Peak frequency $f_p$

The peak frequency is defined as the discrete frequency where the variance density spectrum  $E(f)$  is maximum, so that  $f_p$  is defined as

$$E(f_p) = \max E(f_j) \quad (13)$$

Note  $f_p$  is thus a discrete variable with a constant resolution equal to the spectral resolution.

### Peak Period $T_p$

The peak period is calculated as the inverse of the peak frequency

$$T_p = \frac{1}{f_p} \quad (14)$$

Due to the linear frequency grid, the resolution at which the peak period decreases geometrically. Note that this means that for long period wave the resolution is relatively coarse (e.g. the longest resolvable periods are 25.6 and 34.1 s, a gap of 9.5 s), which can result in large jumps in peak period for modest changes in peak frequency.

### Peak Direction $\theta_p$

The peak direction  $\theta_p$  is measured clockwise in degrees from North and points in the direction the waves come from. It is obtained from the directional moments as [Kuik et al., 1988, their equation 12]

$$\theta_p = 270^\circ - \frac{180^\circ}{\pi} \arctan 2(b_1(f_p), a_1(f_p)) \quad (15)$$

where  $a_1(f_p)$  and  $b_1(f_p)$  are the lowest order directional moments evaluated at the peak frequency  $f_p$ . Note that  $\arctan 2(y, x)$  denotes the multivalued arc-tangent function to resolve the full 360 circle. We ensure that  $0^\circ \leq \bar{\theta} \leq 360^\circ$  by adding/subtracting  $360^\circ$  to angles smaller than  $0^\circ$  or larger than  $360^\circ$ , respectively.

**Peak Directional Spreading**  $\sigma_{\theta_p}$ 

The peak directional spreading is measured in degrees and obtained from the directional moments as [Kuik et al., 1988, their equation 24]

$$\sigma_{\theta_p} = \frac{180^\circ}{\pi} \sqrt{2 \left( 1 - \sqrt{a_1^2(f_p) + b_1^2(f_p)} \right)} \quad (16)$$

where  $a_1(f_p)$  and  $b_1(f_p)$  are the lowest-order directional moments evaluated at the peak frequency  $f_p$ .

## References

- L H Holthuijsen. *Waves in oceanic and coastal waters*. Cambridge Univ. Press, 2007.
- A J Kuik, G P Van Vledder, and L H Holthuijsen. A method for the routine analysis of pitch-and-roll buoy wave data. *J. Phys. Oceanogr.*, 18(7):1020–1034, 1988.