Spotter Technical Reference Manual

Sofar

Abstract

This document describes definitions of (bulk) spectral variables output by Sofar Spotter. In general, our definitions follow standard oceanographic practice, and the specifics are given here. For motivation and/or physical significance of the definitions we refer to the bibliography at the end of this document.

Contents

1 Wave spectrum and directional moments

Spotter measures the wave-induced 3D displacements as East-West ($x(t)$), North-South ($y(t)$ and Up-Down ($z(t)$). By default, Spotter samples displacement data at 2.5 Hz for 30 minutes and computes the cross-spectral matrix. The displacement data and the spectral information is stored on the internal storage (sd-card) and the spectra are used to calculate statistical wave parameters (detailed below) that are transmitted to the Sofar Dashboard.

Specifically, spectral estimates of (cross-)spectral densities are provided on a discretely sampled frequency grid

$$f_j = j\Delta f \qquad\qquad j = 4 \ldots 127 \tag{1}$$

where $\Delta f = f_s/256 \approx 0.0097\,\mathrm{Hz}$. Note that frequencies below $f_4 = 0.029$ are not resolved by Spotter and in the analysis this information is not used (and set to NaN in output on internal storage).

For each discrete frequency, Spotter calculates spectral estimates for the following (all spectra have units m^2/Hz):

Symbol	Spectral Quantity
$E_{zz}(f_j)$	Surface elevation variance density spectrum
$E_{xx}(f_j)$	x-displacement variance density spectrum
$E_{yy}(f_j)$	y-displacement variance density spectrum
$C_{xy}(f_j)$	x–y co-spectrum
$Q_{xz}(f_j)$	x–z quad-spectrum
$Q_{yz}(f_j)$	x–y quad-spectrum

The surface elevation variance density spectrum is used to calculate bulk wave heights and period measures. Directional properties are estimated using the lowest order directional moments of the directional distribution, which are related to the (co/quad-)spectra and displacement spectra as [e.g. Kuik et al., 1988, their appendix A]

$$a_1(f) = \frac{Qxz}{\sqrt{E_{zz}(Exx + Eyy)}} \tag{2}$$

$$b_1(f) = \frac{Qyz}{\sqrt{E_{zz}(Exx + Eyy)}} \tag{3}$$

$$a_2(f) = \frac{Exx - Eyy}{Exx + Eyy} \tag{4}$$

$$b_2(f) = \frac{2Cxy}{Exx + Eyy} \tag{5}$$

where a_n and b_n are the lowest order cosine and sine Fourier series components, respectively.

2 Bulk Parameter definitions

To define wave statistical Bulk parameters we will use various spectral frequency moments, defined as

$$m_n = \int_{f_4}^{f_{127}} E(f) f^n \,\mathrm{d}f \tag{6}$$

where the lower and upper integration limits coincide with the limits of the variance density spectrum $E(f)$. In addition, we define spectrally weighted mean parameter as

$$\overline{X} = (m_0)^{-1} \int_{f_4}^{f_{127}} E(f) X(f) \,\mathrm{d}f \tag{7}$$

From the directional moments a_1, b_1, a_2, b_2 mean directional moments are then defined as

$$\bar{a}_1(f) = \frac{\overline{Q}xz}{\sqrt{\overline{E}_{zz}(\overline{E}xx + \overline{E}yy)}} \tag{8}$$

$$\bar{b}_1(f) = \frac{\overline{Q}yz}{\sqrt{\overline{E}_{zz}(\overline{E}xx + \overline{E}yy)}} \tag{9}$$

$$\bar{a}_2(f) = \frac{\overline{E}xx - \overline{E}yy}{\overline{E}xx + \bar{E}yy} \tag{10}$$

$$\bar{b}_2(f) = \frac{2\overline{C}xy}{\overline{E}xx + \overline{E}yy} \tag{11}$$

where $\bar{a}_1$ (and $\bar{b}_1$ etc) are the mean spectrally-weighted mean directional moments and $\overline{E}_{zz}$ etc. denote mean spectral variables. In all cases the integration of frequency-dependent quantities F(f) is approximated discretely according to the trapazoidal rule.

2.1 Significant Wave height H_{m_0} (meters)

The significant wave height H_{m_0} is estimated from the the variance density spectrum as [e.g. Holthuijsen, 2007]

$$H_{m_0} = 4\sqrt{m_0} \tag{12}$$

with m_0 defined as in equation (6).

2.2 Mean Period T_{m01} (seconds)

A characteristic mean period of the wave field is estimated from the the variance density spectrum according to [e.g. Holthuijsen, 2007]

$$T_{m01} = \frac{m_0}{m_1} \tag{13}$$

with m_0, m_1 defined as in equation (6).

2.3 Mean Direction $\bar{\theta}$ (seconds)

The mean direction $\bar{\theta}$ is measured clockwise in degrees from North and points in the direction the waves come from. It is obtained from the directional moments as [Kuik et al., 1988, their equation 12]

$$\bar{\theta} = 270^\circ - \frac{180^\circ}{\pi} \operatorname{arctan2}\left(\bar{b}_1, \bar{a}_1\right) \tag{14}$$

where $\bar{a}_1$ and $\bar{b}_1$ are the lowest order bulk directional moments (see eq. 8). Note that $\arctan 2(y, x)$ denotes the multivalued arc-tangent function to resolve the full 360 circle. We ensure that $0^\circ \leq \bar{\theta} \leq 360^\circ$ by adding/subtracting 360^0 to angles smaller then 0° or larger than 360^0, respectively.

2.4 Mean Directional Spreading $\bar{\sigma}_\theta$ (degrees)

The mean directional spreading is measured in degrees and obtained from the bulk directional moments as [Kuik et al., 1988, their equation 24]

$$\bar{\sigma}_\theta = \frac{180^\circ}{\pi} \sqrt{2\left(1 - \sqrt{\bar{a}_1^2 + \bar{b}_1^2}\right)} \tag{15}$$

where $\bar{a}_1$ and $\bar{b}_1$ are the lowest-order bulk directional moments (see eq. 8).

2.5 Peak frequency f_p (Hertz)

The peak frequency is defined as the discrete frequency where the variance density spectrum $E(f)$ is maximum, so that f_p is defined as

$$E(f_p) = \max E(f_j) \tag{16}$$

Note f_p is thus a discrete variable with a constant resolution equal to the spectral resolution.

2.6 Peak Period T_p (seconds)

The peak period is calculated as the inverse of the peak frequency

$$T_p = \frac{1}{f_p} \tag{17}$$

Due to the linear frequency grid, the resolution at which the peak period decreases geometrically. Note that this means that for long period wave the resolution is relatively coarse (e.g. the longest resolvable periods are 25.6 and 34.1 s, a gap of 9.5 s), which can result in large jumps in peak period for modest changes in peak frequency.

2.7 Peak Direction θ_p (degrees)

The peak direction θ_p is measured clockwise in degrees from North and points in the direction the waves come from. It is obtained from the directional moments as [Kuik et al., 1988, their equation 12]

$$\theta_p = 270^\circ - \frac{180^\circ}{\pi} \operatorname{arctan2}\left(b_1(f_p), a_1(f_p)\right) \tag{18}$$

where $a_1(f_p)$ and $b_1(f_p)$ are the lowest order directional moments evaluated at the peak frequency f_p. Note that $\arctan 2(y, x)$ denotes the multivalued arc-tangent function to resolve the full 360 circle. We ensure that $0^\circ \leq \bar{\theta} \leq 360^\circ$ by adding/subtracting 360^0 to angles smaller then 0° or larger than 360^0, respectively.

2.8 Peak Directional Spreading σ_{θ_p} (degrees)

The peak directional spreading is measured in degrees and obtained from the directional moments as [Kuik et al., 1988, their equation 24]

$$\sigma_{\theta_p} = \frac{180^\circ}{\pi} \sqrt{2\left(1 - \sqrt{a_1^2(f_p) + b_1^2(f_p)}\right)} \tag{19}$$

where $a_1(f_p)$ and $b_1(f_p)$ are the lowest-order directional moments evaluated at the peak frequency f_p.

3 Partitioned Data

In addition to the bulk parameters, Spotter can provide parameters calculated over two partitions P_1 and P_2. For wave climates that have predictable 'sea' and 'swell' bands this allows reporting of wind sea and swell parameters. To this end, two partitions of the frequency range are defined as:

$$P_1 : \qquad f_4 \leq f < f_{\text{sep}} \tag{20}$$

$$P_2 : \qquad f_{\text{sep}} \leq f < f_{128} \tag{21}$$

with the separation frequency

$$f_{\text{sep}} = j_{\text{swell}} \Delta f, \qquad , 6 \leq j_{\text{swell}} \leq 21 \tag{22}$$

Here P_1 represents the low frequencies (e.g. Swell), and P_2 the high frequencies (e.g. wind Sea). Note that the separation frequency lies between 0.059 and 0.21 Hz. For each of the partitions we now define spectral frequency moments,

$$m_n^k = \int_{P_k} E(f) f^n \, \mathrm{d}f \tag{23}$$

where i denotes the partition number, and the lower and upper integration limits now coincide with the limits of the partition considered. Similarly, we define spectrally weighted mean parameter over the partition as

$$\overline{X}^k = (m_0^i)^{-1} \int_{P_k} E(f) X(f) \, \mathrm{d}f \tag{24}$$

With these definitions in place, the partitioned variables are defined as follows

3.1 Partitioned Significant Waveheight (meters)

For each partition the significant waveheight is defined as:

$$H_{m_0}^k = 4\sqrt{m_0^k} \tag{25}$$

with the moments defined as in (23). The superscript k denotes the number of the partition considered (i.e. P_k is P_1 or P_2).

3.2 Partitioned Mean Period (seconds)

For each partition the mean period is defined as:

$$T_{m_0 1}^k = \frac{m_0^k}{m_1^k} \tag{26}$$

with the moments defined as in (23). The superscript k denotes the number of the partition considered (i.e. P_k is P_1 or P_2).

3.3 Partitioned Mean Direction $\bar{\theta}^k$ (seconds)

The partitioned mean direction $\bar{\theta}$ is measured clockwise in degrees from North and points in the direction the waves come from. It is obtained from the partitioned directional moments as [Kuik et al., 1988, their equation 12]

$$\bar{\theta}^k = 270^\circ - \frac{180^\circ}{\pi} \operatorname{arctan2}\left(\bar{b}_1^k, \bar{a}_1^k\right) \tag{27}$$

where $\bar{a}_1^k$ and $\bar{b}_1^k$ are the lowest order bulk partitioned directional moments defined using (23). Note that $\arctan 2(y, x)$ denotes the multivalued arc-tangent function to resolve the full 360 circle. We ensure that $0^\circ \leq \bar{\theta} \leq 360^\circ$ by adding/subtracting 360^0 to angles smaller then 0° or larger than 360^0, respectively. The superscript k denotes the number of the partition considered (i.e. P_k is P_1 or P_2).

3.4 Partitioned Mean Directional Spreading $\bar{\sigma}_\theta^k$ (degrees)

The mean partitioned directional spreading is measured in degrees and obtained from the partitioned mean directional moments as [Kuik et al., 1988, their equation 24]

$$\bar{\sigma}_\theta^k = \frac{180^\circ}{\pi}\sqrt{2\left(1-\sqrt{\bar{(}a_1^k)^2+\bar{(}b_1^k)^2}\right)} \quad (28)$$

where $\bar{a}_1^k$ and $\bar{b}_1^k$ are the lowest-order partitioned directional moments defined using (23). The superscript k denotes the number of the partition considered (i.e. P_k is P_1 or P_2).

4 References

L H Holthuijsen. *Waves in oceanic and coastal waters*. Cambridge Univ. Press, 2007.

A J Kuik, G P Van Vledder, and L H Holthuijsen. A method for the routine analysis of pitch-and-roll buoy wave data. *J. Phys. Oceanogr.*., 18(7):1020–1034, 1988.